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Instanton of Type IIB Supergravity in Ten Dimensions

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Abstract: A new instanton solution in Type IIB supergravity is presented. This admits the standard interpretation as a tunneling amplitude between classically degenerate ground states. A number of associated issues are discussed, such as the relation to a previously discovered instanton, the Dirac quantization condition, and supersymmetry.

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1. Introduction

Type IIB supergravity (SUGRA) represents the low energy effective field theory describing massless particles with momenta below the scale of massive modes of the superstring. Type IIB SUGRA contains, in addition to the generic fields of the Neveu-Schwarz (NS) sector (the metric tensor $g_{\mu\nu}$, antisymmetric tensor $B_{\mu\nu}$, and dilaton ϕ), gauge potentials characteristic of the Ramond-Ramond (RR) sector (a scalar axion a, two-form $C_{\mu\nu}$, four-form $C_{\kappa\lambda\mu\nu}$, and their duals). These RR potentials couple locally to charges carried by non-perturbative states called D-branes.[1] D-branes may also be seen as solitonic solutions of the source-free classical field equations of the dual (magnetic) form of SUGRA.[2] The D=–1 brane is unique among D-branes since it is localized in time (albeit Euclidean) as well as in space. The interpretation is that it is an instanton, signaling the existence of a non-perturbative transition amplitude of SUGRA. In this note, we present an instanton solution that is more general and more easily interpretable than the one found previously.[3] In fact, the solution found there will be seen to be more like a half-brane, which resolves some of the paradoxes associated therewith.

If the D=–1 brane were like other branes, it would appear as both a magnetic formulation in which it arises as an extended solution of the "source-free" field equations as well as an electric formulation, in which it is postulated as an elementary source coupled locally to the dual potential. In this paper, we present a magnetic description in terms of the C_8 potential. In a companion paper,[4] we offer an electric description in terms of the axion a potential. In fact, for a complete determination of the instanton's parameters as well as its interpretation as a tunneling transition between classically degenerate states, we must refer to this dual description. The idea of an electric description of an instanton is a novel construct, to say the least. Heretofore, nonperturbative tunneling amplitudes could be found only when a semiclassical, or WKB-like approximation, was appropriate. The idea that an instanton can be introduced directly as an elementary object opens up a world of new possibilities for field theories.

The outline of this paper is as follows: In the next section, we review the dual forms of the Lagrangian of interest, paying special attention to the difference between Lorentzian and Euclidean signature. In Sec. 3, we present our instanton solution, and, in Sec. 4, we discuss some of the features of the associated tunneling amplitude, as well as the relation of our solution to the instanton previously discovered.[3] Finally, in Sec. 5, we conclude with a summary and some prospects for further developments.

2. Lorentzian versus Euclidean Signature

The dual forms of the SUGRA Lagrangian with which we shall be concerned are, in 10 dimensions in the Einstein frame,[1]

$$\mathcal{L}_0 = -R + \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}e^{2\phi}{F_1}^2 \tag{2.1}$$

$$\mathcal{L}_8 = -R + \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2\cdot 9!}e^{-2\phi}F_9^2 \tag{2.2}$$

where $F_1 \equiv da$ and $F_9 \equiv dC_8$. The correspondence between the two RR fields is

$$e^{2\phi}F_1 \ = {}^*F_9 \tag{2.3}$$

where * denotes the Hodge or Poincare dual. In eqs. (2.1) and (2.2), we have set to zero all those fields whose classical values vanish. Of course, to determine stability and to calculate correlation functions, they must be resurrected. It is generally assumed that either formulation can be used as a starting point for describing the same physics. The corresponding equations of motion (EOM), *up to the addition of possible source terms,* are

$$\begin{aligned} \nabla_\mu(e^{2\phi}\nabla^\mu a) &= 0 \\ -\nabla^2\phi + e^{2\phi}(\nabla a)^2 &= 0 \\ R_{\mu\nu} &= \tfrac{1}{2}\nabla_\mu\,\phi\,\nabla_\nu\,\phi + \tfrac{1}{2}e^{2\phi}\nabla_\mu\,a\,\nabla_\nu\,a \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} \nabla_\mu(e^{-2\phi}F_9^{\mu\mu_1\ldots\mu_8}) &= 0 \\ \nabla^2\phi + \tfrac{1}{9!}e^{-2\phi}F_9^2 &= 0 \\ R_{\mu\nu} &= \tfrac{1}{2}\nabla_\mu\,\phi\nabla_\nu\,\phi + \tfrac{1}{2\cdot 8!}e^{-2\phi}F_\mu{}^{\mu_1\ldots\mu_8}F_{\nu\mu_1\ldots\mu_8} - g_{\mu\nu}\tfrac{1}{2\cdot 9!}e^{-2\phi}F_9^2 \end{aligned} \tag{2.5}$$

where we have rewritten Einstein's equations using the form of the scalar curvature in each case,

$$R = \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}e^{2\phi}(\nabla a)^2 \tag{2.6}$$

$$R = \tfrac{1}{2}(\nabla\phi)^2 - \tfrac{1}{2\cdot 9!}e^{-2\phi}F_9^2 \tag{2.7}$$

Up to now, we have not specified whether we interpret the preceding discussion to be in a spacetime with Lorentzian signature (LS) or Euclidean signature (ES). It turns out that there are important differences. With LS, the dual forms of the Lagrangian given in eqs. (2.1) and (2.2) do <u>not</u> transform into each other under the formal substitution given in eq. (2.3). This is because of a minus sign that appears for LS

$$\tfrac{1}{9!}e^{-2\phi}{F_9}^2 = -e^{2\phi}{F_1}^2. \tag{2.8}$$

However, one may verify that the EOM <u>are</u> interchanged by the duality transformation.[1].

[1]This observation is in accord with Dirac's original discussion of duality in electrodynamics in four-dimensions.

On the other hand, with ES, the correspondence is reversed, *viz.*, the Euclidean actions do transform into each other but the EOM do not, the crucial difference resulting from the sign flip

$$\tfrac{1}{9!}e^{-2\phi}{F_9}^2 = +e^{2\phi}{F_1}^2. \tag{2.9}$$

For all other D-branes, these observations are irrelevant, since they are static solutions of the EOM and, hence, are independent of the sign of the time. However, for the D=–1 brane, these differences are critical. Indeed, one may justifiably wonder[5] whether the presumed equivalence between the dual formulations of the theory should not be reexamined.

An instanton is used to compute a semiclassical (or WKB) approximation to a tunneling amplitude in field theory. It is usually defined to be a minimum of the Euclidean action, a solution of the EOM having finite action. Extending this definition to quantum gravity is complicated by the fact that the Euclidean scalar curvature is not positive semi-definite. Worse, it is not bounded from below, so it would seem to be hopeless to seek the absolute minimum of the Euclidean classical action. It is not our goal to resolve this controversial issue here, but it seems that we must take a position to proceed at all. We will adopt the point of view (advocated by Hartle and Schleich[6]) that this aspect of the Einstein-Hilbert action, which is associated with the conformal mode, is very likely a kind of gauge artifact and not a physical breakdown in the theory. Regardless of one's view on the ultimate marriage of gravity with quantum mechanics, it is hard to believe that the leading contribution to the effective field theory at distances large compared to the Planck length is not proportional to the scalar curvature. And even if one does not share the view that the Euclidean formulation is fundamental, any discussion of gravitational instantons requires a resolution of this dilemma. Otherwise, it seems as though there would always be infinite tunneling rates!

In the case at hand, it may be seen that, for any stationary configuration, the source-free EOM yields a non-negative value for the action. For this, one need only consider Einstein's equations, which imply that the scalar curvature satisfies eq. (2.7). This implies that the value of the action is

$$S_V = \int d^{10}x\sqrt{g}\ \tfrac{1}{9!}e^{-2\phi}{F_9}^2 \geq 0. \tag{2.10}$$

Since a D=–1 brane is expected to be a source-free solution of the magnetic (C_8) form of the theory, one may conclude that any nontrivial solution of the EOM will yield a strictly positive value for the action.

In the dual formulation in terms of the axion field a, the implication of the source-free Einstein equation eq. (2.6) is that the value of the Lagrangian density eq. (2.1) is always zero. For static D-brane solitons, a classical source would be introduced, coupled "electrically" to the RR-field in order to reproduce the effects found in the dual, "magnetic" formulation. In the present case, the analog would be a point source located at the site of the instanton (usually chosen to be the origin of coordinates) and coupled locally to the axion field at that point. On the other hand, the introduction of a classical source for a transition amplitude that is supposed to be inherent to the theory seems impermissible. The resolution of this paradox will be seen subsequently to be that, like infinity, the origin is actually not part of the background Euclidean spacetime. Like infinity which, as noted in the next section, cannot be compactified, so also the origin cannot be attached to the classical background as if it were a point. In fact, the roles of the origin and infinity are identical and, because of an apparent isometry of the metric, may be interchanged. Thus, the "source" is transformed into a boundary condition at the origin, as required by current conservation. With the origin removed, the topology of the space is qualitatively different, allowing for nontrivial "windings" about the origin. (See eq. (3.3) below.)

Because they determine the boundary conditions on the semiclassical solutions, let us reflect on the classical ground states of this theory. In either formulation, these states correspond to a flat space, $g_{\mu\nu} = \eta_{\mu\nu}$, and a constant value of the dilaton field $\phi(x) = \phi_0$. In the axion formulation, eq. (2.1), the axion field also takes an arbitrary, constant value of the field $a(x) = a_0$. The Type IIB SUGRA action[1] possesses an SL(2,R) global symmetry that is explicitly broken by the Type IIB superstring action,[7], so SL(2,R) is believed to be an "accidental" symmetry of the SUGRA action and expected to be explicitly broken by higher order terms of the effective field theory.[2] Each degenerate classical ground state spontaneously breaks the global SL(2,R) symmetry down to $R^1_{\tau_0}$, where $R^1_{\tau_0}$ denotes those modular transformations that leave $\tau_0 = a_0 + i\exp(-\phi_0)$ invariant. Thus, two of the three generators of SL(2,R) are spontaneously broken, and the corresponding massless modes (or Goldstone bosons in the quantum theory) are just the fluctuations in $\widehat{a} \equiv a - a_0$ and $\widehat{\phi} \equiv \phi - \phi_0$, *i.e.*, these scalar fields are their own Goldstone bosons. In the dual formulation, eqs. (2.2) and (2.5), the ground states have $F_9 = 0$, so that C_8 must be pure gauge, $C_8 = d\Lambda_7$ (including, possibly, zero). The SL(2,R) symmetry is not a Noether symmetry but nevertheless can be seen to be a symmetry of the solutions of the source-free EOM.[8] There remains a global R^1 scaling symmetry that is spontaneously broken for which, once again, the fluctuations in $\widehat{\phi} = \phi - \phi_0$ form the Goldstone mode. The natural expectation for the role of the instanton would be to correspond to tunneling between these distinct ground states.

3. Instanton Solution

3.1. Solving the Euclidean EOM

To solve the field equations eq. (2.5) in Euclidean space, it is natural to make an ansatz

[2] It has been conjectured that there remains an exact SL(2,Z) discrete gauge symmetry.[7]

similar to that made for other D-branes. Motivated by the expectation that the minimal action solution occurs for the most symmetric configuration, we seek an SO(10) invariant solution. We make therefore, the ansätze

$$g_{\mu\nu} = \Omega^2(r)\delta_{\mu\nu}, \qquad \phi = \phi(r), \tag{3.1}$$

where r is the radial coordinate in ten-dimensions. We further assume that the only non-zero component of F_9 is the angular component

$$\text{“}F_9\text{”} = \frac{q_J}{\Omega_9}\omega_9 \tag{3.2}$$

where ω_9 is the volume nine-form on S^9, and $\Omega_9 = 32\pi^4/105$ is the volume of the unit nine-sphere. This implies that F_9 is closed, $dF_9 = 0$, but not exact, *i.e.*, although $F_9 = dC_8$ locally, the potential C_8 is not globally well-defined. To see this, consider any region M the includes the origin $r = 0$. Then it follows from eq. (3.2) that

$$\int_M dF_9 = \int_{\partial M} F_9 = q_J, \tag{3.3}$$

But if $F_9 = dC_8$ for a function C_8, these integrals would vanish. In fact, F_9 is singular at the origin, and it is impossible to find a function C_8 that is nonsingular everywhere on a closed surface (*e.g.*, S^9). This sort of situation is familiar, for example, from discussions of the Dirac monopole.[9][3] A similar discussion applies in a neighborhood of ∞, where one may think of the opposite charge $-q_J$ residing. Assuming that, as $r \to \infty$, the metric becomes flat and ϕ tends to a constant, finiteness of the action eq. (2.10) requires that $F_{\mu_1 \cdots \mu_9}$ fall faster than $O(r^{-5})$ in spherical coordinates. In fact, by Gauss's theorem, the ansatz eq. (3.2) requires that it fall as r^{-9}.

Note that q_J has dimensions of $[length]^8$ and is the only dimensionful parameter encountered thus far.[4] Thus, its value is a matter of convention, and we may choose $q_J = \pm 1$ if we wished. Only its sign is relevant, and it is trivial to transform between from the solution for one sign into the other.

Given these ansätze, it then follows from the first equation in eq. (2.5) that

$$e^{-2\phi} F_9 = {}^*db \tag{3.4}$$

for some scalar field $b = b(r)$. Since the classical ground states have $F_9 = 0$, $b(r)$ should tend to a constant value asymptotically. Then, from eqs. (3.2) and (3.4), it follows that

$$J^r = g^{rr} e^{2\phi} \frac{\partial b}{\partial r} = \frac{q_J}{\Omega_9 \sqrt{g}} \tag{3.5}$$

[3]Unlike monopoles in broken GUTs, in this case, there is no corresponding homotopic argument requiring q_J to be quantized.

[4]Of course, the quantum of action in ten-dimensions, κ^2, would enter the calculation of the transition amplitude, but this parameter does not enter the classical EOM.

Nevertheless, eq. (3.4) together with the Bianci identity, $dF_9 = 0$, imply

$$\nabla_\mu(e^{2\phi}\nabla^\mu b) = 0 \tag{3.6}$$

except possibly at $r = 0$ where F_9 cannot be defined. Thus, the current $J^\mu \equiv e^{2\phi}\nabla^\mu b$ is conserved, except possibly at the origin. In fact, in view of eq. (3.3), it is as if there were a point charge there, since

$$\nabla_\mu J^\mu = \nabla_\mu \left(e^{2\phi}\nabla^\mu b\right) = \frac{q_J \delta^{10}(x)}{\sqrt{g}} \tag{3.7}$$

How is eq. (3.7) compatible with the view that we seek a solution of the <u>source-free</u> EOM eq. (2.5)? This question implicitly requires that we specify the region over which we seek such a solution. Although it is conventional to regard such "magnetic" solutions as non-singular,[2] the fact of the matter is that the corresponding RR potentials, in this case C_8, are simply not well-defined at the "center" of the solution, *i.e.*, at the origin of the transverse coordinates. The behavior of C_8 at the origin, like its behavior at infinity, is a reflection of the fact that it is necessarily singular on any closed surface surrounding the origin. Thus, the background field is not well-defined at the origin of the D-brane. In this sense, the source-free EOM hold everywhere <u>except</u> at $r = 0$ and $r = \infty$. This is however, only half the answer, since the origin certainly is a source of RR charge, just as infinity is a complementary sink. The other half of this story will be explained in the next section: the geometry of the background spacetime is not a simply-connected Riemannian surface but is "cylindrical," $R \times S^9$, with the neighborhood of the origin identical to the neighborhood of infinity.[5] The behaviors at the origin and at infinity correspond to boundary conditions on the fields.

We digress at this point to remark that we are already in a position to determine the <u>value</u> of the action for the instanton, at least formally, even before having solved the remaining equations! Returning to eq. (2.10), we reexpress F_9 in terms of b and then use eq. (3.5))

$$S_V = \int d^{10}x\sqrt{g}e^{2\phi}\left(\nabla b\right)^2 = \int d^{10}x\sqrt{g}\ \nabla_\mu b J^\mu = q_J \int dr \frac{\partial b}{\partial r} = q_J \Delta b, \tag{3.8}$$

where $\Delta b \equiv b(r = \infty) - b(r = 0)$. Note that, by eq. (3.5), b is monotonically increasing (decreasing) with r depending on whether the sign of q_J is positive (negative). Thus, the sign of Δb is the same as the sign of q_J, and so S_V is positive, as it must be according to eq. (2.10). This result eq. (3.8) depends only on the conservation of the current (which is true in the absence of seven-branes) and the net change Δb.[6] It is not at all clear at this

[5] This assumes that the naked singularity discussed in the next subsection is somehow cut off.

[6] Even though the function $b(r)$ is defined by eq. (3.4) only up to an additive constant, the difference Δb should be physically meaningful.

point how Δb should be determined, since the asymptotic condition is merely constant ϕ and $F_9 = 0$. We shall return to this issue in Section 4.

Next, consider the EOM for the dilaton in eq. (2.5). If we insert the solution in terms of b, eq. (3.4), we arrive at the simpler equation

$$\begin{aligned} \nabla^2\phi + e^{2\phi}(\nabla b)^2 = 0, \qquad & \text{or} \\ \nabla_\mu(\nabla^\mu\phi + b e^{2\phi}\nabla^\mu b) = 0 \quad & (r \neq 0,\ \infty) \end{aligned} \tag{3.9}$$

Thus, $K^\mu \equiv \nabla^\mu\phi + bJ^\mu$ is a conserved current (except possibly at $r = 0$ or $r = \infty$).[7] Therefore, introducing another integration constant q_K,

$$\sqrt{g}g^{rr}\frac{\partial\phi}{\partial r} + \frac{q_J}{\Omega_9}b(r) = \frac{q_K}{\Omega_9}, \tag{3.10}$$

where we used eq. (3.5) for J^r. The constant q_K acts like a source for the current K^μ in the same way that q_J appears in eq. (3.7), *viz.*,

$$\nabla_\mu K^\mu = \frac{q_K\delta^{10}(x)}{\sqrt{g}} \tag{3.11}$$

As remarked earlier, rather than an external source at the origin $x^\mu = 0$, we think of the origin as not in the space and the behavior of the fields there as a boundary condition. We have noted previously that the definition of K_μ is arbitrary up to a shift by a constant times J^μ, and, similarly, eq. (3.10) is valid for any b satisfying eq. (3.4). Since b is arbitrary up to a constant, the value of q_K has no intrinsic physical meaning. For example, the value of the action eq. (3.8) is clearly independent of q_K. The physical question is how the fields behave in the presence of the non-zero current J^μ.

The form of eqs. (3.5) and (3.10) suggest the introduction of a new coordinate y defined by

$$-dy \equiv \frac{8\ell^8 dr}{\sqrt{g}g^{rr}}, \tag{3.12}$$

where we have introduced an arbitrary scale ℓ to make y dimensionless. From the ansatz for the metric, we have $\sqrt{g}g^{rr} = r^9\Omega^8$, so that, from eq. (3.12), the relation between y and r may be expressed as

$$\Omega^8 dy = d\left(\frac{\ell}{r}\right)^8 \tag{3.13}$$

Consequently, one finds that $\sqrt{g}g^{yy} = 8\ell^8$, a constant.

[7]Since $b(r)$ is arbitrary up to a constant, the definition of K^μ may correspondingly be shifted by a constant times J^μ. More generally, expressed in terms of C_8, one can show that K^μ is not invariant under gauge transformations of C_8, and so is not a physically observable current density.

Returning to the EOM and changing to the coordinate y, eqs. (3.5) and (3.10) become

$$\begin{aligned} \frac{\partial \widehat{b}}{\partial y} &= -\widetilde{q_J}\, e^{-2\phi} \\ \frac{\partial \phi}{\partial y} &= \widetilde{q_J}\, \widehat{b}, \end{aligned} \tag{3.14}$$

where we defined $\widetilde{q_J} \equiv q_J/8\ell^8\Omega_9$, $\widehat{b} \equiv b - k$, and $k \equiv q_K/q_J$. This equation implies

$$\begin{aligned} \frac{\partial \phi}{\partial \widehat{b}} &= -\widehat{b}e^{2\phi}, \quad \text{so that} \\ \widehat{b}^2 - e^{-2\phi} &= C^2 \end{aligned} \tag{3.15}$$

for some constant C^2. The sign of C^2 is undetermined at this point, but it turns out that, for $C^2 < 0$, the value of the action is ill-defined, despite eq. (3.8), because the solution for b has a nonintegrable singularity at some finite radius. This case is discussed in the Appendix, where we show also that the metric in that case is perfectly regular everywhere. On the other hand, for $C^2 > 0$, b undergoes a finite step at some radius r_s, but, unlike the case $C^2 < 0$, this occurs in a strong coupling region where string corrections are expected to be important, so one may hope for a resolution of the discontinuity from quantum corrections. The curvature is also singular at r_s in the Einstein frame, although we shall exhibit another frame in which the curvature remains finite everywhere. The case $C^2 = 0$, which was considered in ref. [3]will be discussed in due course.

Assuming that $C^2 > 0$, the general solution of eq. (3.14) can then be found

$$\widehat{b} = C \coth(\omega(y + y_0)), \quad Ce^{\phi} = \left|\sinh(\omega(y + y_0))\right|, \text{ where } \omega \equiv \tilde{q}_J C, \tag{3.16}$$

and y_0 is an integration constant. To resolve sign ambiguities, we take $C \geq 0$ and *choose the sign of ω to have the sign of q_J*. Inasmuch as q_K is not physically observable and is tied to the convention for b, there is no loss of generality in choosing $q_K = 0$, so that $\widehat{b} = b - k = b$. We will adopt this convention henceforth.

3.2. Background Geometry

Heretofore, we have not needed the explicit solution for the conformal factor $\Omega(r)$ for the metric. In terms of y, the metric becomes

$$ds^2 = \Omega^2[dr^2 + r^2 d\omega_9^2] = \ell^2 \left(\frac{r\Omega}{\ell}\right)^2 [\left(\frac{r\Omega}{\ell}\right)^{16} \frac{dy^2}{64} + d\omega_9^2] \tag{3.17}$$

As the precise form of the angular measure plays no role in our discussion, we need focus only on the radial dependence. To determine $\Omega(r)$ explicitly, we need to solve Einstein's equations, eq. (2.5), which, in terms of our solution for F_9, can be shown to reduce to

$$R_{\mu\nu} = \tfrac{1}{2}\nabla_\mu\phi\, \nabla_\nu\phi - \tfrac{1}{2}\nabla_\mu b\, \nabla_\nu b \tag{3.18}$$

Given our ansatz for the metric, $R_{\mu\nu}$ may be expressed in terms of Ω as

$$R_{\mu\nu} = -\frac{\delta_{\mu\nu}}{8r^{17}\Omega^8}\frac{\partial}{\partial r}\left(r^{17}\frac{\partial}{\partial r}\Omega^8\right) + \frac{8\Omega}{r}x_\mu x_\nu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\Omega^{-1}\right) \tag{3.19}$$

On the other hand, the right-hand side of eq. (3.18) clearly is proportional to the tensor $x_\mu x_\nu$ only, so that the coefficient of $\delta_{\mu\nu}$ must vanish. This implies[8]

$$\Omega^8 = 1 - \omega_R^2 \left(\frac{\ell}{r}\right)^{16}, \tag{3.20}$$

where, in accord with our notion of the classical ground states, we imposed the condition that the metric be asymptotically flat as $r \to \infty$. Intuitively, we would expect the integration constant $\omega_R^2 < 0$ to avoid a naked singularity, but, as discussed in the Appendix, this leads to other divergences making the action ill-defined. Thus, this solution eq. (3.20) apparently holds only beyond the singularity

$$r > r_s \equiv \ell\omega_R^{\frac{1}{8}}. \tag{3.21}$$

It then follows from eq. (3.13) that

$$\left(\frac{\ell}{r}\right)^8 = \frac{\tanh(\omega_R y)}{\omega_R}, \qquad y \geq 0, \tag{3.22}$$

where, without loss of generality, we chose $y = 0$ to correspond to $r = \infty$. From eq. (3.20), the conformal factor is

$$\Omega^{-4} = \cosh(\omega_R y). \tag{3.23}$$

To satisfy Einstein's equation, we must also determine that the $x_\mu x_\nu$ term in eq. (3.19) agrees with the right-hand-side of eq. (3.18). This is most simply expressed in terms of the coordinate y as

$$R_{yy} = \frac{9}{2}\Omega^4\frac{\partial^2}{\partial y^2}\Omega^{-4} = \tfrac{1}{2}\left(\frac{\partial\phi}{\partial y}\right)^2 - \tfrac{1}{2}e^{2\phi}\left(\frac{\partial b}{\partial y}\right)^2. \tag{3.24}$$

The right-hand-side is, according to eqs. (3.14) and (3.15), given by

$$\tfrac{1}{2}\tilde{q}_J^2\left(b^2 - e^{-2\phi}\right) = \tfrac{1}{2}\tilde{q}_J^2\, C^2 \equiv \frac{\omega^2}{2} \geq 0. \tag{3.25}$$

Therefore, eq. (3.24) reduces to

$$\frac{\partial^2}{\partial y^2}\Omega^{-4} = \left(\frac{\omega^2}{9}\right)\Omega^{-4}. \tag{3.26}$$

[8]Note that the form of the solution cannot be guessed by naively extrapolating from higher D-branes.[2]

Comparing with eq. (3.23), we see that Einstein's equations are satisfied provided we take[9]

$$\omega_R = \frac{|\omega|}{3}. \tag{3.27}$$

The corresponding scalar curvature takes the form

$$R = \frac{32\omega^2}{\ell^2}\left(\frac{\ell}{r\Omega}\right)^{18} \geq 0. \tag{3.28}$$

The nontrivial coordinate dependence in the metric and the curvature involves the single quantity

$$\left(\frac{\ell}{r\Omega}\right)^8 = \frac{\sinh(2\omega_R|y|)}{2\omega_R} = \frac{1}{\omega_R}\left|\left(\frac{r_s}{r}\right)^8 - \left(\frac{r}{r_s}\right)^8\right|^{-1}, \tag{3.29}$$

Thus, in the Einstein frame, the curvature is everywhere nonnegative, and the metric is asymptotically flat as $r \to \infty$ ($y \to 0+$). On the other hand, the curvature diverges as $r \to r_{s}+$ ($y \to +\infty$). The result in eq. (3.29) is valid only for $y > 0$ ($r > r_s$) but will be extended shortly to $y < 0$ ($r < r_s$).

Inasmuch as the SUGRA action is the leading term in a derivative expansion of the effective action, this divergence of the curvature suggests that these EOM break down as $r \to r_s$. However, because this is not a purely metric theory of gravity, it sometimes happens in such cases that the metric is well-behaved in another "frame" associated with a conformal rescaling of the metric by the dilation, *i.e.*,

$$\begin{aligned} \widetilde{g_{\mu\nu}} &\equiv e^{p\phi}\, g_{\mu\nu} = e^{p\phi}\, \Omega^2 \delta_{\mu\nu}, \\ \widetilde{ds}^2 &= e^{p\phi}\, ds^2. \end{aligned} \tag{3.30}$$

So we need to determine the form of $\exp(\phi)$. The generic form of the solutions for the other fields is given in eq. (3.16). To adapt them to the present situation, the subsequent discussion is somewhat simplified if we take $\omega > 0$, but it can be easily translated for the case $\omega < 0$. To simplify writing, *we shall assume $\omega > 0$ throughout the remainder of this paper.* Then the solution is

$$b = C\coth(\omega(y + y_+)),\ \ e^{\phi} = C^{-1}\sinh(\omega(y + y_+)),\ \ y \geq 0,\ \text{with } e^{\phi_+} \equiv C^{-1}\sinh(\omega y_+) > 0, \tag{3.31}$$

where, for future reference, we have also recorded the solution for b. The integration constant y_+ must be positive to avoid further singularities at finite y. Then $\exp(\phi)$ is finite for $y \geq 0$, but $\phi \to \omega y$ as $y \to +\infty$ ($r \to r_{s}+$). From eq. (3.23), $\Omega \to \exp(\omega y/12)$, so that

$$e^{p\phi}\, \Omega^2 \to e^{(p-\frac{1}{6})\omega y} \text{ as } y \to +\infty.$$

[9] Since the metric in the Einstein frame is SL(2,R) invariant, ω_R (and, hence, ω) are SL(2,R) invariant.

Therefore, we expect that, for $p = 1/6$, the background curvature in this frame will be finite as $r \rightarrow r_s$. Because the metric is conformally flat, all information about the curvature is contained in the Ricci tensor. A straightforward calculation shows that $\widetilde{R_{\mu\nu}} dx^\mu dx^\nu$ is indeed finite, nonzero as $r \rightarrow r_s$. In this frame, which will be referred to as the instanton frame,[10] the scalar curvature turns out to be

$$\widetilde{R} = \frac{2}{C^2} e^{-\frac{13}{6}\phi} R, \tag{3.32}$$

so that $\widetilde{R} \sim |r - r_s|$ as $r \rightarrow r_{s+}$. Thus, rather than diverging, the scalar curvature vanishes at the singularity, so only the traceless part of the Ricci tensor is nonvanishing there. In the instanton frame, the space is geodesically complete, and nothing prevents us from proceeding to the region $r < r_s$.[11]

The behavior of the curvature in the instanton frame suggests that it should be possible to connect the exterior solution for $r > r_s$ to an interior solution for $r < r_s$. However, because the local string coupling $\exp(\phi)$ diverges as $r \rightarrow r_s$, the semiclassical approximation breaks down, regardless of frame, so one cannot be sure. Nevertheless, away from the singularity, we can elaborate an interior solution that is essentially the mirror image of the exterior solution. Returning to the Einstein frame, the analog of eq. (3.20) is

$$\Omega^8 = \omega_R^2 \left(\frac{\ell}{r}\right)^{16} - 1, \qquad r < r_s. \tag{3.33}$$

In principle, we could entertain the possibility that ω_R^2 is different than in the exterior region, but if we wish the singularity to occur for the same value of r_s and for the same "instanton" frame to have a nonsingular curvature at r_s, we must take ω_R^2 to have the same value as in the exterior region. (Other reasons, such as current conservation, will be seen below.) Solving eq. (3.13) once again for the relation between the coordinates r and y, we find

$$\left(\frac{\ell}{r}\right)^8 = -\frac{\coth(\omega_R y)}{\omega_R}, \qquad y \leq 0. \tag{3.34}$$

At the risk of some confusion, we chose the origin $r = 0$ to correspond to $y \rightarrow 0-$, although we could of course have chosen any other convenient value as well.[12] Thus, $y \rightarrow -\infty$ corresponds to the approach $r \rightarrow r_{s-}$ to the singularity from the interior. From eq. (3.20), the conformal factor turns out to be

$$\Omega^{-4} = -\sinh(\omega_R y), \qquad y \leq 0. \tag{3.35}$$

[10] As noted earlier, many of the formulas in Ref. [2] cannot be applied directly to the p=–1-brane.

[11] The "instanton frame" also has the property that the kinetic energy for the dilaton vanishes, so its EOM becomes an equation of constraint.

[12] Since $y \rightarrow 0+$ corresponds to $r \rightarrow \infty$, these two limits $y \rightarrow 0\pm$ correspond to opposite ends of the space.

As before, the radial part of Einstein's equations will then be satisfied provided ω_R and ω are related by eq. (3.27). The scalar curvature in the Einstein frame is again given by eqs. (3.28) and (3.29). Since the metric in terms of the coordinate y, given in eq. (3.17), depends only on the combination in eq. (3.29), the space is asymptotically flat as $r \to 0$. Formally, the metrics in the interior and exterior regions are related by the replacement $y \to -y$ $(r \to r_s^2/r)$. The metric in the instanton frame also manifests this inversion symmetry (provided also $y_+ \leftrightarrow -y_-$) and is nonsingular everywhere, it is natural to conjecture that this isometry persists despite the singularity.

Analogous to eq. (3.31), the corresponding interior solutions for b and ϕ are

$$b = -C\coth(\omega(y_- - y)), e^{\phi} = C^{-1}\sinh(\omega(y_- - y)), y \le 0, \text{ with } e^{\phi_-} \equiv C^{-1}\sinh(\omega y_-) > 0. \tag{3.36}$$

where, again, $|\omega| = 3\omega_R$. Note that the behavior of $\exp(\phi)$ and Ω near the singularity is the same as in the exterior region, so that the same transformation eq. eq. (3.30) from the Einstein frame to the instanton frame removes the singularity. Even though b undergoes a jump from $-C$ to $+C$ in crossing the singularity at $r = r_s$, the value of the action is still given by eq. (3.8), with

$$\Delta b = C\left(\coth(\omega y_+) + \coth(\omega y_-)\right) > 2C. \tag{3.37}$$

Note that $\nabla_\mu J^\mu = 0$ at the singularity (most easily seen by noting that $J_y = -q_J$ is constant), so that current conservation holds across the singularity so long as q_J is the same in the interior and exterior regions. The asymptotic values of the dilaton field are related to the string couplings in the initial and final states $g_\mp \equiv \exp(\phi_\mp)$. It remains to determine the values of $b_\pm$ or, more precisely, of Δb. From eq. (3.15), we have

$$C^2 = b_+^2 - e^{-2\phi_+} = b_-^2 - e^{-2\phi_-}. \tag{3.38}$$

Therefore, we may express C in terms of the string couplings and Δb as

$$C = \frac{1}{2|\Delta b|}\Lambda\left(\Delta b^2, e^{-2\phi_+}, e^{-2\phi_-}\right)^{\frac{1}{2}} = \frac{|\Delta b|}{2}\Lambda\left(\frac{e^{\Delta\phi}}{\Delta b'^2}, \frac{e^{-\Delta\phi}}{\Delta b'^2}, 1\right)^{\frac{1}{2}}, \tag{3.39}$$

where Λ is the "triangle function" defined as $\Lambda(x,y,z) \equiv x^2+y^2+z^2-2xy-2yz-2xz$, and, in the second expression, we abbreviated $\phi_+ - \phi_- \equiv \Delta\phi$ and $\Delta b^2 \exp(\phi_+ + \phi_-) \equiv \Delta b'^2$. We have argued in the preceding that we must have $C^2 \ge 0$, but the function $\Lambda(x,y,z)$ is not positive for all values of its arguments. If we regard the asymptotic values of the string couplings as fixed, this provides a constraint on the allowed range of Δb. This can be seen most easily by writing

$$4C^2 e^{\phi_+ + \phi_-} \equiv 4C'^2 = \frac{1}{\Delta b'^2}\left[\left(\Delta b'^2 - 4\cosh^2\frac{\Delta\phi}{2}\right)\left(\Delta b'^2 - 4\sinh^2\frac{\Delta\phi}{2}\right)\right]. \tag{3.40}$$

The requirement that $C^2 > 0$ *excludes* the range

$$2 \sinh \frac{|\Delta\phi|}{2} < |\Delta b'| < 2 \cosh \frac{\Delta\phi}{2}. \tag{3.41}$$

We shall return below to the question of just how Δb is to be determined.

4. Further Considerations about Instanton Properties

4.1. Determination of Δb

As discussed at the conclusion of the Introduction, we expect the instanton to represent tunneling between asymptotic states associated with a flat metric, definite values of $\phi_\pm$, and $F_9 = 0$. However, to uniquely specify the solution of the classical field equations for the instanton, we must know the change Δb in the asymptotic values of the associated field b, values that are not determined by the requirement $F_9 = 0$. Therefore, it is tempting to assume that $|\Delta b|$ takes its smallest allowed value, since that would correspond to the minimum allowed value of the action. Such a choice corresponds to the limiting case $C = 0$ (or $\omega = 0$), the case that was treated in ref. [3].[13] Naively, in the limit $\omega \to 0$ (for fixed $q_J \neq 0$), $r_s \to 0$, according to eq. (3.21), so that the singularity ends up at the origin. The interior region seems to shrink to a point, and the space apparently becomes flat everywhere (except at the origin). Formally, $C = 0$ corresponds to $|\Delta b'|$ equal to the boundary values of the range specified in eq. (3.41). Regardless of which root is chosen, this involves two string couplings, $g_\pm = \exp(\phi_\pm)$, and suggests the persistence of both regions. Moreover, because of the apparent isometry $r \to r_s^2/r$, the roles of the interior and exterior regions might well be exchanged, so that it is the outer region that disappears in this limit. From the present point of view, the instanton described in [3] is more like half an instanton, and its interpretation as a tunneling amplitude is obscure.

How then do we propose to determine Δb? In ref. [4], we argue that the proper correspondence is to determine Δb by reference to the corresponding value of Δa in the dual picture, where $\Delta a \equiv (a_+ - a_-)$, with $a_\mp$, the initial and final values of the axion field.[14] The values q_J, ω_R, and ω are taken to be the same as in the present treatment. Accordingly, we expect the instanton presented here to give the minimal action in semiclassical approximation for the transition between certain states $|\phi_-, a_-\rangle$ and $|\phi_+, a_+\rangle$. Given the values of $\Delta\phi$ and Δa, the value of C' may be determined. Then, given C', eq. (3.40) may be inverted to determine $|\Delta b'|$. Of course, there are two real roots of the quadratic equation eq. (3.40), so further information is needed to resolve this two-fold ambiguity. Note that, from eqs. (3.31) and (3.36), we must have $|\Delta b| > 2C$, and, generically, only the larger value of $|\Delta b|$ will fulfill this constraint. However, for C'

[13] More precisely, in ref. [3], only a single asymptotic value of the string coupling is specified.

[14] This is quite different from attempts to identify b somehow with a or ia.[3]

sufficiently small, both roots do fulfill this condition, so a further argument is needed to exclude the smaller root. It turns out that there is a lower limit on C set by the smaller of the asymptotic string couplings, which may be expressed as $C'_{min} = \exp(|\Delta\phi|/2)$.[15] This lower limit is above the value at which the smaller root comes into the allowed range, so it is always the larger value of $|\Delta b|$ that must be selected. Thereby, Δb is uniquely determined by the corresponding Δa, their signs being determined to agree with the sign of the charge q_J. Similarly, the reverse correspondence may be used to associate a given value of Δb with a transition between states with a certain Δa.

This lower limit on C specifically excludes the case treated in [3], protecting us against some of the paradoxes of that construction. However, although different in detail, our results have much in common with the philosophy of ref. [3], where the hope was expressed that string effects would resolve the apparent singularity. Further, it was suggested by reference to the string frame metric, that, in fact, there should be an underlying inversion symmetry of the sort discussed here in the Einstein frame and that the neighborhood of the origin somehow represented another asymptotic spacetime. For reasons explained earlier, we share those same beliefs, but assume that the resolution of this naked singularity will permit the usual association of the instanton with a tunneling amplitude between classically degenerate states. Unfortunately, like so many other arguments involving D-branes and duality, we cannot prove this in the semiclassical approximation but can only appeal to the overall self-consistency of the description.

4.2. Dirac Quantization?

Another important question concerns whether the charge q_J must be quantized because of the Dirac condition on the relation between the charges of the instanton and the seven-brane. Dirac's argument that the product of electric and magnetic charge must be quantized was used in Ref. [3] to conclude that the product of q_J with the seven-brane charge must be quantized, an argument that was used much earlier by Teitelboim and collaborators[11] for dual point charges. The result is based on the single-valuedness of the wave function of a charge when moved about its dual, owing to the existence of a "Dirac string" or its analog for whatever dimensional object under consideration. Even though universally accepted,[1] we feel there is reason to doubt this conclusion for instantons. As emphasized above, the collective coordinates associated with the instanton, such as its location, must be integrated over to form the transition amplitude for tunneling. Unlike static solitons, which exist in spacetime with Lorentzian signature, there are no phases in the Euclidean path integral, and one cannot speak of a wave function for an instanton, let alone "move a seven-brane about a transition amplitude" as envisioned in such proofs. We believe that the appropriateness of such arguments remain to be

[15] The lower limit is achieved for $|\Delta a| = |\exp(-2\phi_+) - \exp(-2\phi_-)|$ or, equivalently, $|\Delta a'| = 2\sinh|\Delta\phi|$.

demonstrated for instantons and, until persuaded otherwise, we do not assume that the charge q_J is quantized.[16] In this regard, it is extremely interesting that the seven-brane is unique among D-branes, inasmuch as its charge quantization does **not** depend on the existence of the dual charge.[3, 12] This lack of reliance of the seven-brane charge on the dual charge reinforces our skepticism about instanton charge quantization.

4.3. Instantons and Supersymmetry

An important issue concerns the supersymmetric nature of the instanton and whether the D=–1-brane preserves some subset of the original supersymmetries, as do other D-branes. This is quite problematic, since the formulation of supersymmetry in Euclidean spacetime is at best, ambiguous, and at worst, ill-defined, especially for theories having either chiral or Majorana fermions.[13] In fact, the real question is whether the instanton leads to a tunneling transition amplitude that preserves the supersymmetries of the theory with Lorentzian signature. Because supersymmetry is gauged in SUGRA, the answer to this question must be a resounding "yes" if the gauge symmetry is to remain nonanomalous. One may think that, since SUGRA is nonrenormalizable anyway, this is not so important as in ordinary, vector gauge theories, but that is not the case. Regardless of renormalizability, the associated Ward identities implied by supersymmetry can be preserved only if the gauge symmetries are anomaly-free. Demonstrating that this is the case for the D=–1-brane may be quite informative, as it is in the case of supersymmetric QCD,[14, 15] but the manner in which it comes about must be analogous. Unlike other D-branes, which are infinitely massive, the collective coordinates of the instanton must be integrated over in calculating the transition amplitude. These collective coordinates, associated with zero modes, include both bosonic ones, such as the location of the center of the instanton solution, and fermionic ones or Grassman collective coordinates. Regardless of which supersymmetries might be broken by a given instanton background, after integration they must be restored for the tunneling amplitude.[17] To state the same thing another way, the exact effective action, including the nonperturbative effects of tunneling, would still have the complete set of gauged supersymmetries (at least in an appropriate background gauge). Of course, one could imagine that global SUSY were spontaneously broken by the exact ground state, but that is another matter.

5. Summary and Conclusions

To summarize the picture that has emerged, the coordinate r is like a Euclidean time

[16] This is an important respect in which we disagree with all earlier literature.

[17] In SUSY QCD, they are actually restored to the integrand after proper definition of the measure of the path integral.[14]

in which $r \to 0$ corresponds to the distant past and $r \to \infty$, to the distant future. Correspondingly, we expect tunneling between states characterized by constant values of b and ϕ. Unfortunately, the semiclassical approximation for the instanton solution breaks down at some intermediate "time." Nevertheless, arguments from string theory suggest the existence of such a $D = -1$-brane, so there is reason to hope that this naked singularity will be cured by quantum corrections. The value of the instanton action eq. (2.10) is independent of the metric singularity, but complicated by the discontinuity in b, since $b \to \pm C$ as $r \to r_s \pm$. Thus, b undergoes a step at r_s, but it is natural to expect that the effect of higher order corrections will be to smooth out the jump so that b will vary smoothly from $-C$ to $+C$ in passing across the region near r_s. Since $\Delta b = b_+ - b_- > 2C$, the naive value for the action in eq. (2.10) makes sense despite this discontinuity in b. Note that $\exp(-2\phi) \sim |r - r_s|^{\frac{3}{2}}$ near the singularity, so there is formally no problem integrating across the singularity to obtain the value of the action given by eq. (3.8).[18]

Near the singularity, the curvature is large in the Einstein frame but remains finite in the instanton frame. This is only suggestive, since $\exp(\phi)$ is large near the singularity so, from the point of view of string theory, this remains a strong coupling regime regardless of frame. While a naked singularity is to be feared, this one occurs only in Euclidean spacetime as an obstacle to the semiclassical description of a tunneling amplitude. We know of no argument suggesting that it is anything more than a breakdown in the perturbative, weak-coupling approximation to the underlying string theory. It might be amusing to determine the $O(\alpha')$ corrections to the SUGRA Lagrangian, regardless of the magnitude of $\exp(\phi)$, to see what effect they have on the singularity. The effects could be quite dramatic, since such corrections are expected to explicitly break the SL(2,R) symmetry. This breaking of current conservation might account for the large change in the scalar field b near the singularity.

We have not attempted the calculation of the transition amplitude itself by carrying out the path integration. We expect that the normalizability of zero modes and others will be plagued by the apparent singularity in the strong coupling region. However, considerable insight about the form of the transition amplitude may be gleaned by exploiting SL(2,R) symmetry.[8]

It would be interesting to obtain this instanton from an eleven-dimensional or M-theory point of view. Because the Type-IIB superstring cannot be obtained directly by dimensional reduction from eleven-dimensional SUGRA, we do not understand the relation of D-branes in 10-dimensions to the structure of SUGRA in 11-dimensions. Perhaps by first reducing to nine dimensions, it would be possible to gain a better understanding of the M-theory connection.

Finally, tunneling between states $|\phi, a\rangle$ associated with constant values of the moduli ϕ

[18] This is in contrast to the case discussed in the Appendix in which, although the spacetime is non-singular, the action integrand is non-integrable.

and a implies that these cannot be the candidate ground states for Type IIB SUGRA because correlation functions will not respect cluster decomposition.[16] The correct ground states will be certain superpositions of these, suggesting the rather radical idea that the ground state does not have fixed string coupling. This and related considerations will be discussed subsequently.[17]

We have noted earlier that the instanton frame is one in which the kinetic energy of the dilaton vanishes in the SUGRA action. Since that is a property of the NS-sector only, this is common to all SUGRA theories. Why this is the frame in which the instanton metric is nonsingular remains to be explicated. The dilation EOM becomes a constraint equation in this frame, from which the dilaton field may be expressed in terms of the other fields of the theory. However, second derivatives of the dilaton field enter Einstein's equation in frames other than the Einstein frame, so if one inserts the solution for the dilaton field, one obtains a very complicated, higher derivative theory. More work on sorting out the dynamics of this frame may be revealing.

Finally, even though the presentation here is given in 10-dimensions, the generic sort of construction presented in this paper will apply in dimensions other than 10 in which the background field content is similar, whether in compactified supergravity models or in other interesting field theories.

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6. Appendix – The case $\omega^2 < 0$.

When solving Einstein's equations, we chose the asymptotically-flat solution with a naked singularity eq. (3.20) rather than the apparently more natural solution

$$\Omega^8 = 1 + \omega_R^2 \left(\frac{\ell}{r}\right)^{16} . \tag{6.1}$$

To treat this case, many of the formulas in the body of the text can be carried over simply by replacing $\omega_R \to i\omega_R$ (and $\omega \to i\omega$). However, the associated replacement of hyperbolic with circular trigonometric functions leads to some dramatic changes. For example, eq. (3.22) becomes

$$\left(\frac{\ell}{r}\right)^8 = \frac{\tan(\omega_R y)}{\omega_R}, \tag{6.2}$$

so the range of y is over one period, *e.g.*, $0<\omega_R y<\pi/2$, corresponding to $\infty>r>0$. Because $|\omega|=3\omega_R$, this corresponds to $0<|\omega|y<3\pi/2$. While the background geometry is perfectly regular, this leads to problems for the solutions for b and ϕ ,

$$\widehat{b}=C\cot(\omega y+\theta), \quad Ce^{\phi}=|\sin(\omega y+\theta)| \tag{6.3}$$

where θ is an integration constant (defined modulo π). Because of the range spanned by $\omega\, y$, it is unavoidable that $\widehat{b}$ and $\exp(-\phi)$ have a singularity at some radius (*cf.* eq. (3.16)). In contrast to the case treated in text, this singularity corresponds to a place where the curvature is finite and where the string coupling vanishes. As a result, there is no reason to expect the leading SUGRA Lagrangian or the semiclassical approximation to break down here. Unfortunately, this singularity is non-integrable and renders the action integral eq. (3.8) ill-defined. One may attempt to define this by analytic continuation in θ, but we have been unable to convince ourselves that this is a sensible procedure.

Another reason to be skeptical about the case $\omega_R^2<0$ is presented in Ref. [4], where it can be seen that the only nonzero component of the Ricci tensor R_{yy} in the Einstein frame is necessarily nonnegative. Thus, the dual description requires $\omega^2>0$ and does not permit $\omega^2<0$, as considered in this Appendix. Therefore, if there is any chance for a sensible field theoretic treatment of the D=-1-brane of string theory, it must be in the case treated in text, despite its apparent singularity and breakdown in the semiclassical approximation.

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